A Computationally Efficient Method for Improving the Speed of Full Wave Analysis CAD Programs for Microstrip Circuits

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Abstract

Moment method analysis of microstrip structures is becoming increasingly popular because of its ability to model very general structures. One drawback is the amount of computer time required. A new, and highly efficient, approximation to the Green's function used in many formulations is presented here.

Introduction

In this paper, new approximations to the Green's functions used in full wave microstrip circuit analysis programs are presented. The approximations have a number of advantages. They rely on the substrate of the circuit being electrically thin, which is a reasonable assumption in realistic circuits. There is no restriction on lateral separation between two points. If the substrate is thin enough, the approximation presented here can be used instead of having to calculate the Green's functions numerically. If the substrate is thicker, the form of the approximation can still be used; the coefficients are determined numerically at a few points by numerical evaluation of the Green's functions. Typically, fewer than 20 points are needed. Another advantage of this representation is that they can be used to speed up the run time of the matrix fill when several frequencies are needed. This results in substantial savings of time in the CAD program.

Typically, several minutes of computer time are required to analyze a single component of a circuit using a full wave analysis CAD program. The problem quickly becomes unreasonable if several data points are required at different frequencies. The time to carry out a computation consists of three parts: the evaluation of the Green's functions involved, the calculation of the individual elements of the matrix, and the solution of the matrix equation. For realistically sized problems, the filling of the matrix can take a majority of the time.

Green's Functions

The Green's functions considered in this paper have been used in the formulation of an integral equation (EFIE or MPIE) for the microstrip problem [1,2], and can be represented as Sommerfeld integrals:

$$G_A(\rho) = \frac{\mu_0}{2\pi}\int_0^\infty dk_\rho J_0(k_\rho\rho)\frac{k_\rho}{D_{TE}}$$

$$G_V(\rho) = \frac{1}{2\pi\epsilon_0}\int_0^\infty dk_\rho J_0(k_\rho\rho)k_\rho\frac{u_0+u\tanh(uh)}{D_{TE}D_{TM}}$$

where:

$$D_{TE} = u_0 + u\coth(uh)$$

$$D_{TM} = \epsilon_r u_0 + u\tanh(uh)$$

and u_0 and u are functions of the integration variable k_ρ:

$$u_0 = \sqrt{k_\rho^2 - k_0^2}$$

$$u = \sqrt{k_\rho^2 - \epsilon_r k_0^2}$$

Approximations

The uniform asymptotic approximations used here are:

(1)
$$\frac{4\pi}{\mu_0}G_A(\rho) \approx \frac{e^{-jk_0\rho}}{\rho} - \frac{e^{-jk_0\sqrt{\rho^2+4h^2}}}{\sqrt{\rho^2+4h^2}}$$

and

(2)
$$\begin{aligned}2\pi\epsilon_0 G_V(\rho) \approx & -\frac{h^2k_0^2(\epsilon_r-1)^2}{\epsilon_r^2\rho}e^{-jk_0\rho} + \frac{1-\eta}{2}\left[\frac{1}{\rho}e^{-jk_0\rho}\right. \\ & \left.-(1+\eta)\sum_{i=1}^{\infty}(-\eta)^{i-1}\frac{1}{R_i}e^{-jk_0R_i}\right] \\ & + 2\pi jA(1-\epsilon_r)e^{-j\alpha}erfc(\sqrt{\alpha}e^{-j\pi/4}) \\ & + surface\ wave\ term\end{aligned}$$

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where h is the substrate thickness, ρ is the radial distance between two points on the air dielectric interface, k_0 is the free space wave number, $\alpha = k_0 \rho m$, and

$$\eta = (\epsilon_r - 1)/(\epsilon_r + 1)$$

$$m = (h^2(k_0^2 - k^2)^2)/(2\epsilon_r^2 k_0)$$

$$A = \frac{hk_0^2 m}{2\epsilon_r}\sqrt{2/\pi k_0 \rho}$$

See [4] for the derivation. These approximations are valid to $O[(k_0 h)^2]$. In fact it has been determined in numerical studies that $k_0 h < .05$ must hold for the approximation to work well for values of $\epsilon_r \approx 12.9$. Many problems fall outside of this range, for example, MMIC's at millimeter wave frequencies and microstrip patch antennas. For these cases, the approximation for G_A is still accurate for small ρ, while the approximation for G_V is accurate for large ρ. We correct this problem by observing that the form of equations (1) and (2) is correct, but that the coefficients become inaccurate. The idea is to multiply certain terms by a function of ρ, which is numerically determined at a few points, and interpolating for any other points that are needed.

For example, the corrected version of G_A is:

$$\frac{4\pi}{\mu_0} G_A \approx \left\{1 - f_A(\rho)\left[1 - \left(\frac{\tan(\Delta)}{\Delta}\right)^2\right]\right\}\left[\frac{e^{-\jmath k_0 \rho}}{\rho} - \frac{e^{-\jmath k_0 R_1}}{R_1}\right]$$

where $f_A(\rho)$ is a numerically determined function, $R_1 = \sqrt{\rho^2 + 4h^2}$, and $\Delta = k_0 h\sqrt{\epsilon_r - 1}$. The form of this expression was arrived at by realizing that for small ρ, equation (1) is exact, while for large ρ

$$\frac{4\pi}{\mu_0} G_A \approx \left(\frac{\tan(\Delta)}{\Delta}\right)^2\left[\frac{e^{-\jmath k_0 \rho}}{\rho} - \frac{e^{-\jmath k_0 R_1}}{R_1}\right]$$

is exact. This is a natural extension of equation (1); f_A must approach one as $\rho \to \infty$, and zero as $\rho \to 0$. It is necessary to evaluate equation G_A numerically so that $f_A(\rho)$ can be calculated for a few values of ρ. f_A can then be computed for any value of ρ using a spline interpolation. The number of times the numerical integration (a very expensive calculation) must be carried out depends on the accuracy required. A good method for carrying out this numerical integration is given in [3].

A similar procedure can be carried out for G_V. In this case, the first term on the right hand side of equation (2) is the source of error. It must be included for $\rho >> h$, but is simply wrong for $\rho \to 0$. One solution is to rewrite equation (2) as

$$\begin{aligned} 2\pi\epsilon_0 G_V(\rho) \approx & -f_V(\rho)\frac{h^2 k_0^2(\epsilon_r - 1)^2}{\epsilon_r^2 \rho} e^{-jk_0\rho} + \frac{1-\eta}{2}\left[\frac{1}{\rho}e^{-\jmath k_0 \rho} - \right. \\ & \left.(1+\eta)\sum_{i=1}^{\infty}(-\eta)^{i-1}\frac{1}{R_i}e^{-\jmath k_0 R_i}\right] \\ & + 2\pi j A(1-\epsilon_r)e^{-\jmath\alpha} erfc(\sqrt{\alpha}e^{-\jmath\pi/4}) \\ & + surface\ wave\ term \end{aligned}$$

where again $f_V(\rho)$ must be determined numerically. The behavior of this correction function is qualitatively similar to f_A; f_V must approach one as $\rho \to \infty$, and zero as $\rho \to 0$.

It is worthwhile to separate G_V and G_A into real and imaginary parts. This is because the only contribution to the imaginary part is the portion of the integral along the branch cut for G_A, while the residue at the surface wave pole must also be included for G_V. Thus it is possible to develop a power series about $\rho = 0$, where the coeficients must be determined numerically.

References

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